Chapter 5

Discrete Probability Distributions

Warmup p.188 #2

- The mean pulse rate for adult women is 77.5 bpm, with a standard deviation of 11.6 bpm. Using the range rule of thumb, would a pulse rate of 100 beats per minute be considered unusual? Explain.
- For the pulse rates of adult women, is a pulse rate of 50 bpm unusual? Explain.
- For a couple having eight children, is it unlikely to have all girls?
- For a couple having three children, is it unlikely to have all girls?

Chapter 4 review #16

- Suppose an athletic team is playing an upcoming three games series. They have a 60% chance of winning their first game, a 75% chance of winning their second, and a 40% chance of winning the third game.
  a) Probability that they win all three games
  b) Probability that they win at most 1 game
  c) Probability that they win at least one game

Distribution

- Frequency Distribution
- Histogram
- Probability Distribution
- Probability Histogram

Definitions

- Random Variable – a variable that has a single numerical value for each outcome of a procedure.
- Probability Distribution – gives the probability for each value of the random variable. Usually expressed as a table, graph, or formula.
- Discrete random variable – collection of values that is finite or countable
- Continuous random variable – infinitely many values, the collection is not countable (not for chapter 5)

Probability Distribution

1. There is a numerical random variable x and its values are associated with corresponding probabilities.
2. $\sum P(x) = 1$
3. $0 \leq P(x) \leq 1$

Probability Distributions for number of girls in 2 births

Number of girls x	$P(x)$
0	0.25
1	0.5
2	0.25

Meets the criteria to be a probability distribution

Poll: Subjects asked if marijuana should be legal

Responses	$P(x)$
Yes	0.41
No	0.52
Don't Know	0.07

Does not meet the criteria of a probability distribution

Executives asked when Job applicants should discuss salary

Number of interviews	$P(x)$
1	0.30
2	0.26
3	0.10

Does not meet the criteria of a probability distribution

Probability Distributions for number of girls in 4 births

Possible outcomes

$bbbb$

$gbbb$
$bgbb$
$bbgb$
$bbbg$

$ggbb \quad bbgg$
$gbgb \quad bggb$
$gbbg \quad bgbg$

$gggb$
$ggbg$
$gbgg$
$bggg$

$gggg$

Number of girls x	$P(x)$
0	
1	
2	
3	
4	

Probability Distributions for number of girls in 4 births

Possible outcomes

$bbbb$

$gbbb$
$bgbb$
$bbgb$
$bbbg$

$ggbb \quad bbgg$
$gbgb \quad bggb$
$gbbg \quad bgbg$

$gggb$
$ggbg$
$gbgg$
$bggg$

$gggg$

Number of girls x	$P(x)$
0	$\frac{1}{16}$
1	$\frac{4}{16}$ or $\frac{1}{4}$
2	$\frac{6}{16}$
3	$\frac{4}{16}$ or $\frac{1}{4}$
4	$\frac{1}{16}$

Probability Distributions for number of girls in 4 births

Possible outcomes

bbbb

gbbb
bgbb
bbgb
bbbg

ggbb *bbgg*
gbgb *bggb*
gbbg *bgbg*

gggb
ggbg
gbgg
bggg

gggg

Number of girls x	$P(x)$
0	.0625
1	.25
2	.375
3	.25
4	.0625

Parameters of a Probability Distribution

- Mean

$$\mu = \sum [x \cdot P(x)]$$

- Variance 1

$$\sigma^2 = \sum [(x - \mu)^2 \cdot P(x)]$$

- Variance 2

$$\sigma^2 = \sum [x^2 \cdot P(x)] - \mu^2$$

- Standard Deviation – square root of variance

Probability Distributions for number of girls in 4 births

Number of girls x	$P(x)$
0	.0625
1	.25
2	.375
3	.25
4	.0625

$$\mu = 0 \cdot .0625 \quad + \quad 1 \cdot .25 \quad + \quad 2 \cdot .375 \quad + \quad 3 \cdot .25 \quad + \quad 4 \cdot .0625$$

$$\boldsymbol{\mu = 2}$$

Probability Distributions for number of girls in 4 births

Number of girls x	$P(x)$
0	.0625
1	.25
2	.375
3	.25
4	.0625

$$v = (0-2)^2 \cdot .0625 + (1-2)^2 \cdot .25 + (2-2)^2 \cdot .375 + (3-2)^2 \cdot .25 + (4-2)^2 \cdot .0625$$

$$\boldsymbol{v = 1}$$

$$\boldsymbol{\sigma = 1}$$

Probability Distributions for number of girls in 4 births

Number of girls x	$P(x)$
0	.0625
1	.25
2	.375
3	.25
4	.0625

$$v = [0^2 \cdot .0625 + 1^2 \cdot .25 + 2^2 \cdot .375 + 3^2 \cdot .25 + 4^2 \cdot .0625] - 2^2$$

$$\boldsymbol{v = 1}$$

$$\boldsymbol{\sigma = 1}$$

Probability Distributions an athletic team playing a 3 game series

Chances of winning
Game 1 – 60%
Game 2 – 75%
Game 3 – 40%

$P(www) = .18$

$P(\overline{w}\overline{w}\overline{w}) = .06$

$P(w\overline{w}\overline{w}) = .09$
$P(\overline{w}w\overline{w}) = .18$
$P(\overline{w}\overline{w}w) = .04$

$P(\overline{w}ww) = .12$
$P(w\overline{w}w) = .06$
$P(ww\overline{w}) = .27$

Number of wins	$P(x)$
0	.06
1	.31
2	.45
3	.18

Probability Distributions an athletic team playing a 3 game series

Chances of winning
Game 1 – 60%
Game 2 – 75%
Game 3 – 40%

Number of wins	$P(x)$
0	.06
1	.31
2	.45
3	.18

$$\mu = 0 \cdot .06 \quad + \quad 1 \cdot .31 \quad + \quad 2 \cdot .45 \quad + \quad 3 \cdot .18$$

$$\boldsymbol{\mu = 1.75}$$

Probability Distributions an athletic team playing a 3 game series

Chances of winning

Game 1 – 60%

Game 2 – 75%

Game 3 – 40%

Number of wins	$P(x)$
0	.06
1	.31
2	.45
3	.18

$$v = [0^2 \cdot .06 \quad + \quad 1^2 \cdot .31 \quad + \quad 2^2 \cdot .45 \quad + \quad 3^2 \cdot .18] - 1.75^2$$

$$\boldsymbol{v = 0.6675}$$

$$\boldsymbol{\sigma = 0.817}$$

Expected Value

- Note: Theoretical probability v. Experimental
- The use of μ in place of $\bar{x}$
- The use of σ in place of s
- E is the expected value and it is the mean value of the outcomes.

5-2 Homework

- p206 #5-11, 15-18

Flipping Coins

- From section 4-2
- Find the probability that when you flip a coin 1000 times, you will get exactly 500 heads
- Find the probability that when you flip a coin 1000 times, you will get exactly 520 heads
- Find the probability that when you flip a coin 1000 times, you will get exactly 480 heads

- Find the probability that when you flip a coin 1000 times, you will get <u>more than</u> 520 heads

- Find the probability that when you flip a coin 1000 times, you will get exactly 500 heads

Arrange HHTTHTHHTHTHH….HHTH with 500 heads

$$\frac{1000!}{500!\,500!}$$

Sample Space 2^{1000}

$$\approx 0.0252$$

- Find the probability that when you flip a coin 1000 times, you will get exactly 520 heads

Arrange HHTTHTHHTHTHH….HHTH with 520 heads

$$\frac{1000!}{520!\,480!}$$

Sample Space 2^{1000}

$$\approx 0.0113$$

- Find the probability that when you flip a coin 1000 times, you will get exactly 480 heads

Arrange HHTTHTHHTHTHH....HHTH with 480 heads

$$\frac{1000!}{480!\,520!}$$

Sample Space 2^{1000}

$$\approx 0.0113$$

- Find the probability that when you flip a coin 1000 times, you will get <u>more than</u> 520 heads

$$P(520) = 0.0113$$
$$P(521) = 0.0104$$
$$P(522) = 0.00959$$
$$P(523) = 0.00876$$
$$P(524) = 0.00798$$
$$\ldots$$
$$P(1000) = 0+$$

$$= COMBIN(1000, G2)/2^{1000}$$

G	H
x	p(x)
521	0.010448506
522	0.009587805
523	0.008762851
524	0.00797687
525	0.007232362
526	0.006531125
527	0.005874295
528	0.005262389
529	0.004695364
530	0.004172673
531	0.003693326
532	0.003255959
533	0.00285889
534	0.002500191
535	0.002177736
536	0.001889267
537	0.00163244
538	0.001404869
539	0.001204173
540	0.001028007
541	0.000874091
542	0.000740236
543	0.000624361
544	0.000524509
545	0.000438855
546	0.000365713
547	0.000303535

979	1.4785E-258
980	3.1683E-260
981	6.4593E-262
982	1.2498E-263
983	2.2885E-265
984	3.9536E-267
985	6.4222E-269
986	9.77E-271
987	1.3858E-272
988	1.8234E-274
989	2.2125E-276
990	2.4583E-278
991	2.4806E-280
992	2.2506E-282
993	1.8132E-284
994	1.2769E-286
995	7.6997E-289
996	3.8653E-291
997	1.5508E-293
998	4.6617E-296
999	9.3326E-299
1000	9.3326E-302
	0.097383164

5.2 Homework (part 2)

P208 #12-14, 19-22

Chapter 5 Section 3

BINOMIAL PROBABILITY DISTRIBUTIONS

BINOMIAL PROBABILITY DISTRIBUTIONS

- The procedure has a fixed number of trials.
- The trials must be independent.
- Each trial must have all outcomes classified into two categories (success and failure).
- The probability of success remains the same in each trial.

FAMILY OF 8 CHILDREN

- $x = number\ of\ girls$
- $P(x)\ is\ the\ probability\ of\ having\ x\ girls$
- $n = number\ of\ trials$
  - $n = 8$
- $P(S) = probability\ of\ succes\ on\ one\ trial$
  - $P(S) = .5$
- $P(F) = probability\ of\ failure\ on\ one\ trial$
  - $P(F) = .5$

FAMILY OF 8 CHILDREN

- $P(3)\ is\ the\ probability\ of\ 3\ girls$
- $P(3) = \frac{56}{256}\ or\ 0.219$

Or

$$P(3) = \frac{8!}{3!\,5!} \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right)$$

How are these the same thing?

SHORTER NOTATION

- $P(3) = \frac{8!}{3!5!} \cdot \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^5$

CHOOSE 8 PEOPLE TO BE IN A CLUB

- *From a population of 1000 people, 600 girls & 400 boys*
- $x = number\ of\ girls$
- $P(x) is\ the\ probability\ of\ selecting\ x\ girls$
- $n = number\ of\ trials$
  - $n = 8$
- $P(S) = probability\ of\ succes\ on\ one\ trial$
  - $P(S) = .6$
- $P(F) = probability\ of\ failure\ on\ one\ trial$
  - $P(F) = .4$

CHOOSE 8 PEOPLE FROM A POPULATION

- $P(3)\ is\ the\ probability\ of\ 3\ girls$
- $P(3) = \frac{8!}{3!5!} \cdot (.6)^3 \cdot (.4)^5$

- $P(4) =$
- $P(2) =$
- $P(1) =$
- $P(0) =$

PROBABILITY DISTRIBUTIONS AN ATHLETIC TEAM PLAYING A 3 GAME SERIES

Chances of winning
Game 1 – 60%
Game 2 – 75%
Game 3 – 40%

$P(www) = .18$

$P(\overline{w}\overline{w}\overline{w}) = .06$

$P(w\overline{w}\overline{w}) = .09$
$P(\overline{w}w\overline{w}) = .18$
$P(\overline{w}\overline{w}w) = .04$

$P(\overline{w}ww) = .12$
$P(w\overline{w}w) = .06$
$P(ww\overline{w}) = .27$

Number of wins	$P(x)$
0	.06
1	.31
2	.45
3	.18

Is this distribution a binomial distribution?
Why?

A TEST WITH 6 MULTIPLE CHOICE QUESTIONS

- Each question has one correct answer (a,b,c,d)
- $P(S) = .25$
- $P(F) = .75$

Number correct	$\boldsymbol{P(x)}$
0	
1	
2	
3	
4	
5	
6	

A TEST WITH 6 MULTIPLE CHOICE QUESTIONS

- Each question has one correct answer (a,b,c,d)
- $P(S) = .25$
- $P(F) = .75$

Number correct	$P(x)$
0	0.178
1	0.356
2	0.297
3	0.132
4	0.033
5	0.004
6	0.0002

WEATHER

- Every day this week has a 30% chance of rain
- $P(S) = .3$
- $P(F) = .7$
- $P(4)\ is\ the\ probability\ that\ it\ will\ rain\ 4\ of\ 7\ days$
- $P(4) = \frac{7!}{4!3!} \cdot (.3)^4 \cdot (.7)^3$
- $= 0.097$

WEATHER

- Every day this week has a 30% chance of rain
- $P(S) = .3$
- $P(F) = .7$

Number days with rain	$P(x)$
0	
1	
2	
3	
4	0.097
5	
6	
7	

WEATHER

- Every day this week has a 30% chance of rain
- $P(S) = .3$
- $P(F) = .7$

Number days with rain	$P(x)$
0	0.0824
1	0.247
2	0.318
3	0.227
4	0.097
5	0.025
6	0.0036
7	0.00022

MEAN, VARIANCE, & STANDARD DEVIATION?

Number correct	$P(x)$
0	0.178
1	0.356
2	0.297
3	0.132
4	0.033
5	0.004
6	0.0002

Number days with rain	$P(x)$
0	0.0824
1	0.247
2	0.318
3	0.227
4	0.097
5	0.025
6	0.0036
7	0.00022

MEAN, VARIANCE, STANDARD DEVIATION? NUMBER OF GIRLS IN 8 TRIALS

POPULATION 1000, 600 GIRLS, 400 BOYS

Number of girls	$P(x)$
0	0.000655
1	0.00786
2	0.0413
3	0.124
4	0.232
5	0.279
6	0.209
7	0.0896
8	0.0168

5-3 HOMEWORK

- p218 #13, 15-24, 35, 39

Chapter 5 Section 4

PARAMETERS FOR BINOMIAL DISTRIBUTION

Reminder

- Probability Distribution describes the *Population*
- mean and standard deviation are parameters, not statistics
- μ *and* σ

For Binomial Distributions

$$\mu = np$$
$$\sigma^2 = npq$$
$$\sigma = \sqrt{npq}$$

$$n = the\ number\ of\ trials$$
$$p = probability\ of\ succes$$
$$q = probability\ of\ failure$$

Or

$$\sigma = \sqrt{np(1-p)}$$

Green eyes 17%

- The percentage of the population with green eyes is 17%
- In a room with 25 students, find the expected mean number with green eyes.

$$\mu = 25 \cdot 0.17$$
$$\mu = 4.25$$

Green eyes 17%

- For the same population, find the standard deviation of students with green eyes

$$\sigma^2 = 25 \cdot 0.17 \cdot 0.83$$
$$\sigma^2 = 3.53$$
$$\sigma = 1.88$$

Green eyes 17%

- Use the range rule of thumb to determine the minimum usual and maximum usual number of students with green eyes in a room with 25 students

$$\mu + 2\sigma = 8.01$$
$$\mu - 2\sigma = 0.49$$

Left Handers 10%

- The percentage of the population that is left handed is 10%
- In a room with 150 teachers, find the expected mean number left handed teachers.

$$\mu = 150 \cdot 0.10$$
$$\mu = 15$$

Left Handers 10%

- For the same population, find the standard deviation of left handed teachers

$$\sigma^2 = 150 \cdot 0.1 \cdot 0.9$$
$$\sigma^2 = 13.5$$
$$\sigma = 3.67$$

Left Handers 10%

- Use the range rule of thumb to determine the minimum usual and maximum usual number of left handed teachers in a room.

$$\mu + 2\sigma = 22.34$$
$$\mu - 2\sigma = 7.66$$

Gender Selection

- Some couples prefer to have baby girls because the mothers are carriers of a X-linked recessive disorder that can be inherited by their sons but none of their daughters

Ericsson Method

- The Ericsson method of gender selection has a 75% success rate. Suppose 100 couples use the Ericsson Method with a result that among 100 babies, there are 75 girls.

- Is the Ericsson method successful

Ericsson method

- Assume that the Ericsson method has no effect and boys and girls are equally likely.

$$n = 100 \qquad p = 0.5 \qquad q = 0.5$$

$$\mu = 50$$
$$\sigma^2 = 25$$
$$\sigma = 5$$

$$\mu + 2\sigma = 60$$
$$\mu - 2\sigma = 40$$

Is 75 girls in 100 births unusual?

5-4 Homework

- p226 #5-7, 9, 11, 12, 15, 16, 20