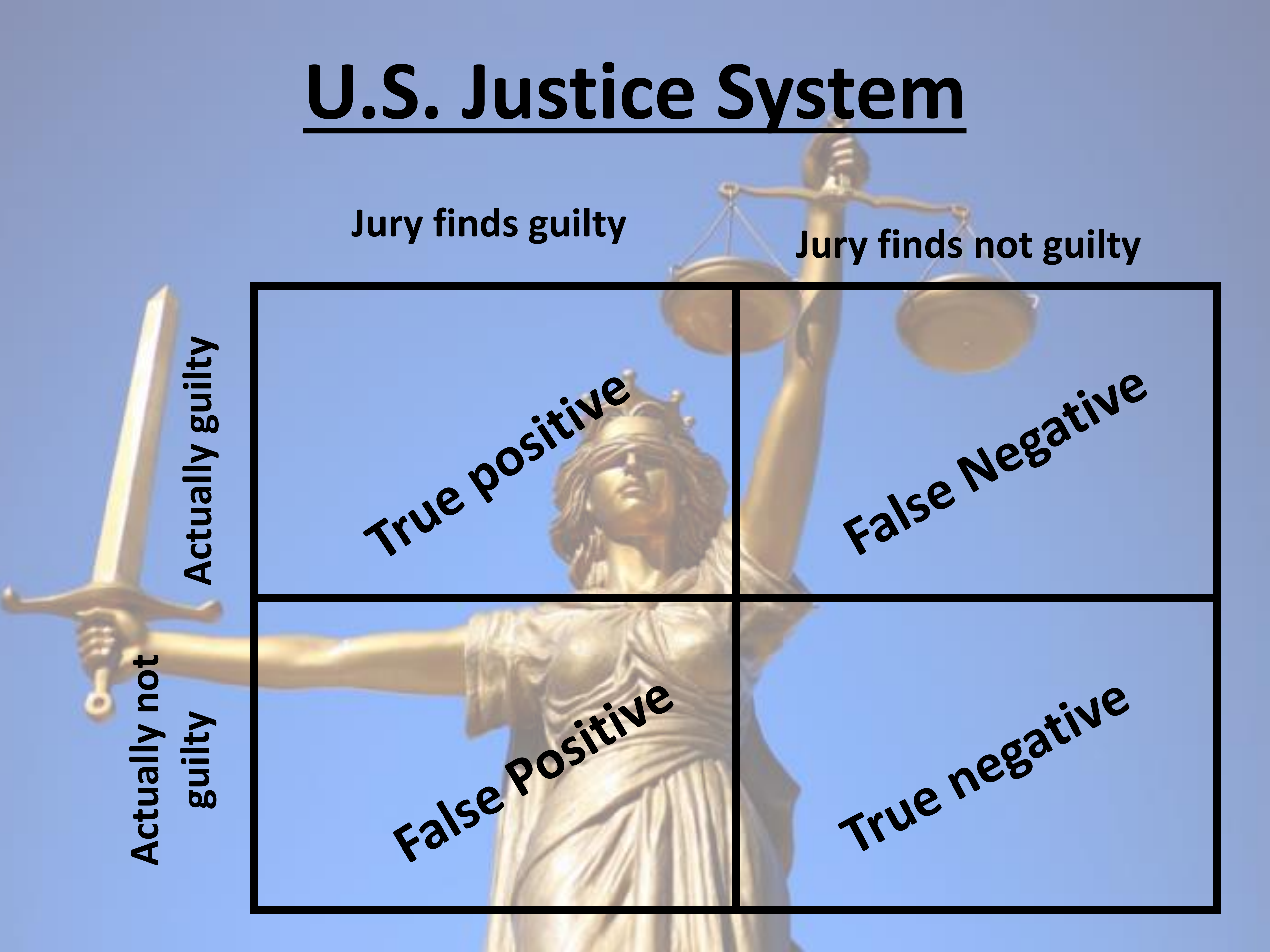
U.S. Justice System



	Jury finds guilty	Jury finds not guilty
Actually guilty	True positive	False Negative
Actually not guilty	False Positive	True negative

- False Positive – Wrong result in which the jury correctly indicates guilt when the defendant is not actually guilty (innocent).
- False Negative – Wrong result in which the jury indicates the defendant is not guilty, when in fact they are guilty.
- True Positive – Correct result in which the jury finds the defendant guilty and they are guilty.
- True Negative – Correct result in which the jury finds the defendant not guilty and they are in fact not guilty (innocent).

Drug Screening

	Positive drug test	Negative drug test
Subject uses drugs	True positive	False Negative
Subject does not use drugs	False Positive	True negative

- False Positive – Wrong result in which the test incorrectly indicates the presence of a condition when the subject does not actually have that condition
- False Negative – Wrong result in which the test incorrectly indicates that the subject does not have a condition when the subject actually does have that condition.
- True Positive – Correct result in which the test correctly indicates that a condition is present when it really is present.
- True Negative – Correct result in which the test correctly indicates that a condition is not present when it really is not present.

Drug Screening

	Positive drug test	Negative drug test
Subject uses drugs	True positive 44	False Negative 6
Subject does not use drugs	False Positive 90	True negative 860

Basic Concepts of Probability

- Event – any collection of results or outcomes of a procedure
- Simple event – an outcome or an event that cannot be further broken down into simpler components
- Sample Space – for a procedure consists of all possible simple events.

Basic Concepts of Probability

- $0 \le P(A) \le 1$
- Complement $1 - P(A)$

Deck of Cards

- **<u>Procedure</u>**
  - I draw one card
- Example of **<u>EVENT</u>**
  - The ace of spades
  - The queen of diamonds
- **<u>Sample Space</u>**
  - List of all events {ace of spades, queen of diamonds, 2 of hearts, 3 of clubs…}

Deck of Cards

- **Procedure**
  - I draw two cards
- Example of **EVENT**
  - The ace of spades & queen of diamonds
  - 2 of hearts and 3 of clubs
  - 2 of hearts and queen of diamonds
- **Sample Space**
  - List of all events {The ace of spades & queen of diamonds, 6 of hearts and 9 of spades…}

Rolling dice

- **Procedure**
  - I roll two dice
- Example of **EVENT**
  - 3 + 4
  - 1 + 6
  - 4 + 3
- **Sample Space**
  - List of all events {3+4, 1+6, 4+3…}

Smoke alarm

- **<u>Procedure</u>**
  - I test the smoke alarm
- Example of **<u>EVENT</u>**
  - Alarm (positive)
  - No alarm (negative)
- **<u>Sample Space</u>**
  - List of all events {positive, negative}

Smoke alarm

- **<u>Procedure</u>**
  - I test the smoke 20 times
- Example of **<u>EVENT</u>**
  - ppppppppnppppppppppp
  - pnppppppppppppppppnp
- **<u>Sample Space</u>**
  - List of all events {pppnppppnnpppppppppp, pppppppppppppppppppp}

Three approaches to probability

- **Relative frequency approximation** – conduct or observe a procedure and count the number of times the "event" occurred.
- **Classical Approach to probability** – (requires equally likely outcomes) Count the number of ways an event can occur, count the number of outcomes in the sample space. $\mathrm{P}(A) = \frac{s}{n}$

Experimental v. Theoretical Probability

$$P(A) = \frac{number\ of\ times\ A\ occured}{number\ of\ times\ the\ procedure\ was\ repeated}$$

$$P(A) = \frac{number\ of\ ways\ A\ occurs}{number\ of\ different\ simple\ events}$$

Procedure – flipping coins

- Probability when we flip a coin three times that we get TAILS, TAILS, TAILS
  - Experimental Design
    - Flip a coin 3 times
    - Count the number of trials, Count the number of Trials with TTT
  - Theoretical Design
    - Count the number of outcomes in the sample space
    - Count the number of outcomes in the event

National League

	OME	AWAY	RS	RA	DIFF	STRK	L10	POFF
x -- LAD	-21	41-34	832	591	+241	L1	6-4	100.0%
z -- ATL	-30	46-30	816	713	+103	W1	5-5	100.0%
STL	-29	37-38	715	616	+99	W2	5-5	95.3%
WSH	-31	41-37	800	687	+113	L1	5-5	94.0%
MIL	-32	37-38	715	736	-21	W1	8-2	62.3%
CHC	-27	31-44	778	664	+114	L3	5-5	41.4%
NYM	-31	36-42	740	703	+37	W2	7-3	6.0%
PHI	-35	35-38	732	736	-4	L1	5-5	0.1%
ARI	-36	39-39	764	709	+55	W1	3-7	0.1%
SF	-42	41-37	656	726	-70	L1	5-5	0.1%
CIN	-35	32-46	675	668	+7	W2	6-4	0.0%
SD	-40	34-44	659	746	-87	L1	3-7	0.0%
COL	-38	26-49	792	910	-118	L2	6-4	0.0%
PIT	-44	34-44	722	866	-144	L6	3-7	0.0%
MIA	-49	24-50	572	751	-179	L1	2-8	0.0%

American League

	OME	AWAY	RS	RA	DIFF	STRK	L10	POFF
z -- HOU	-20	42-33	870	611	+259	W5	7-3	100.0%
x -- NYY	-23	45-31	905	702	+203	W1	6-4	100.0%
MIN	-34	50-25	885	714	+171	W1	6-4	99.8%
OAK	-28	42-33	809	656	+153	W2	8-2	95.3%
TB	-32	47-31	732	622	+110	W1	6-4	54.6%
CLE	-31	43-32	725	603	+122	W4	8-2	50.3%
BOS	-41	43-31	845	770	+75	W1	4-6	0.1%
TEX	-33	32-46	757	809	-52	L5	4-6	0.0%
LAA	-39	33-45	735	819	-84	L1	2-8	0.0%
CHW	-39	31-47	659	800	-141	W1	4-6	0.0%
SEA	-42	32-46	739	866	-127	W5	7-3	0.0%
TOR	-44	31-47	685	777	-92	W4	7-3	0.0%
KC	-47	27-51	647	818	-171	L3	3-7	0.0%
BAL	-55	26-49	676	942	-266	L4	3-7	0.0%
DET	-54	24-53	557	863	-306	L3	3-7	0.0%

Subjective Probability

- Probability you'll die in a plane crash
- Probability you'll have to use the Heimlich Maneuver
- Getting struck by lightning
- Being Mauled by a polar bear and a regular bear on the same day

Homework

- P145 #8, 21-28, 31, 37-40

Probability

4-3 The Addition Rule

Compound Event

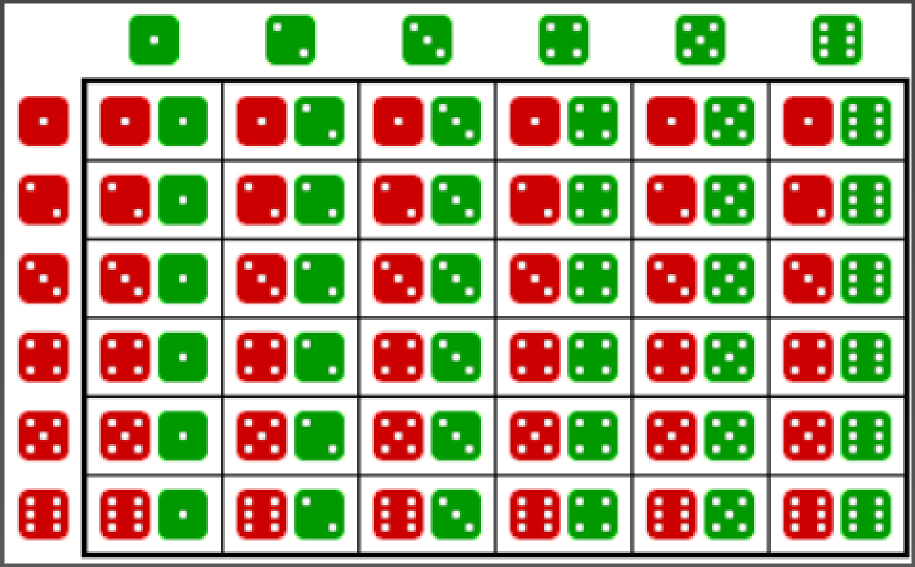
- Compound Event – combining two or more simple events
- What is the probability that even A or Event B occurs
  - $P(A\ or\ B)$

- $P(A) + P(B)???????$
- Not Always

$P(positive\ test\ result\ or\ the\ subject\ uses\ drugs)$

	Positive drug test	Negative drug test
Subject uses drugs	True positive 44	False Negative 6
Subject does not use drugs	False Positive 90	True negative 860

Dice



- Event
  - Roll one die
- $P(2\ or\ 6)$
- $P(2\ or\ an\ even\ number)$

- Event
  - Roll two dice
- $P(\text{red shows 2 or green shows 5}$

Playing Cards

- Event
  - Drawing one card
- $P(A\ black\ card\ or\ a\ red\ card)$
- $P(a\ Queen\ or\ a\ Spade)$

Disjoint Events (Mutually Exclusive)

- Events A and B are **Disjoint** (or mutually exclusive) if they cannot occur at the same time.
  - Disjoint examples
    - Selecting someone who is a registered democrat, selecting someone who is a registered republican
  - *Not* Disjoint
    - Selecting someone who is taking statistics, selecting someone who is female.

Not Disjoint

- $P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

Complimentary Events

- The probability that A occurs
  - $P(A)$
- The probability that A does not occur
  - $P(\bar{A})$

- $P(A) + P(\bar{A}) = 1$

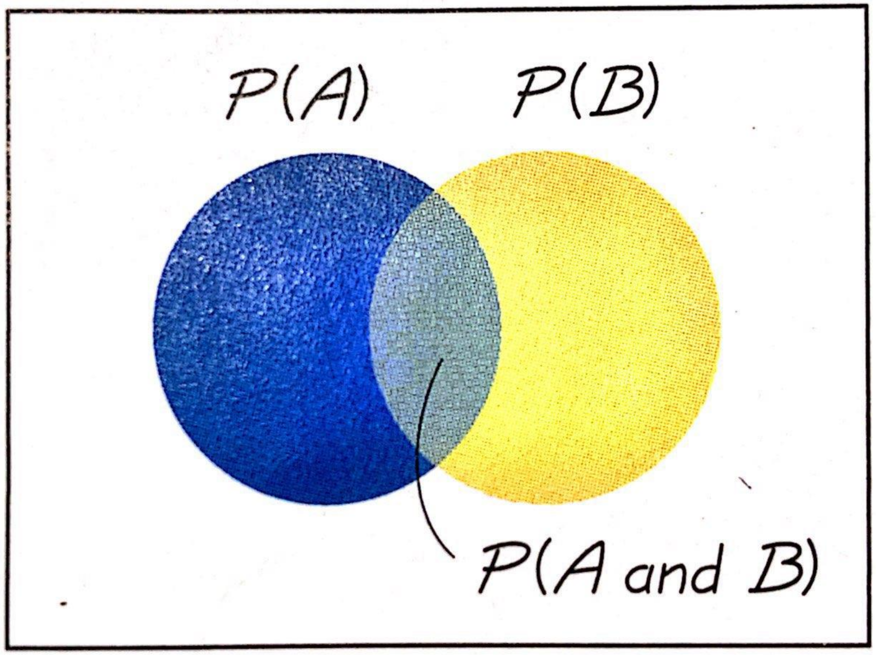


Figure 4-4 **Venn Diagram for Events That Are Not Disjoint**

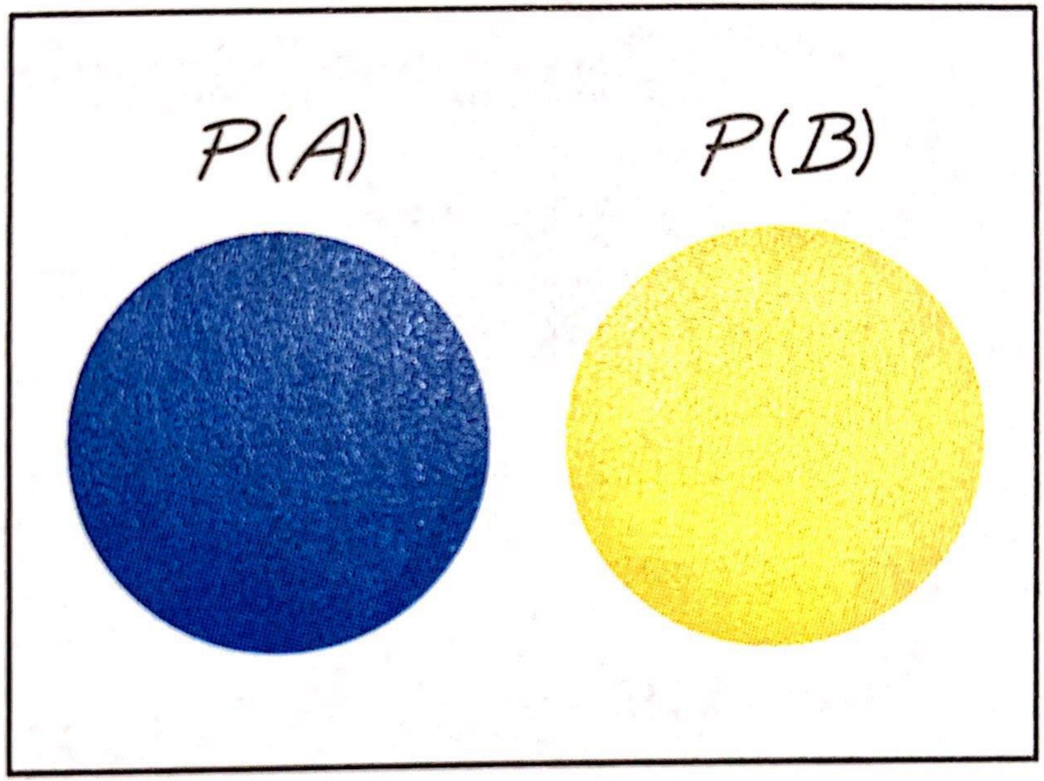


Figure 4-5 **Venn Diagram for Disjoint Events**

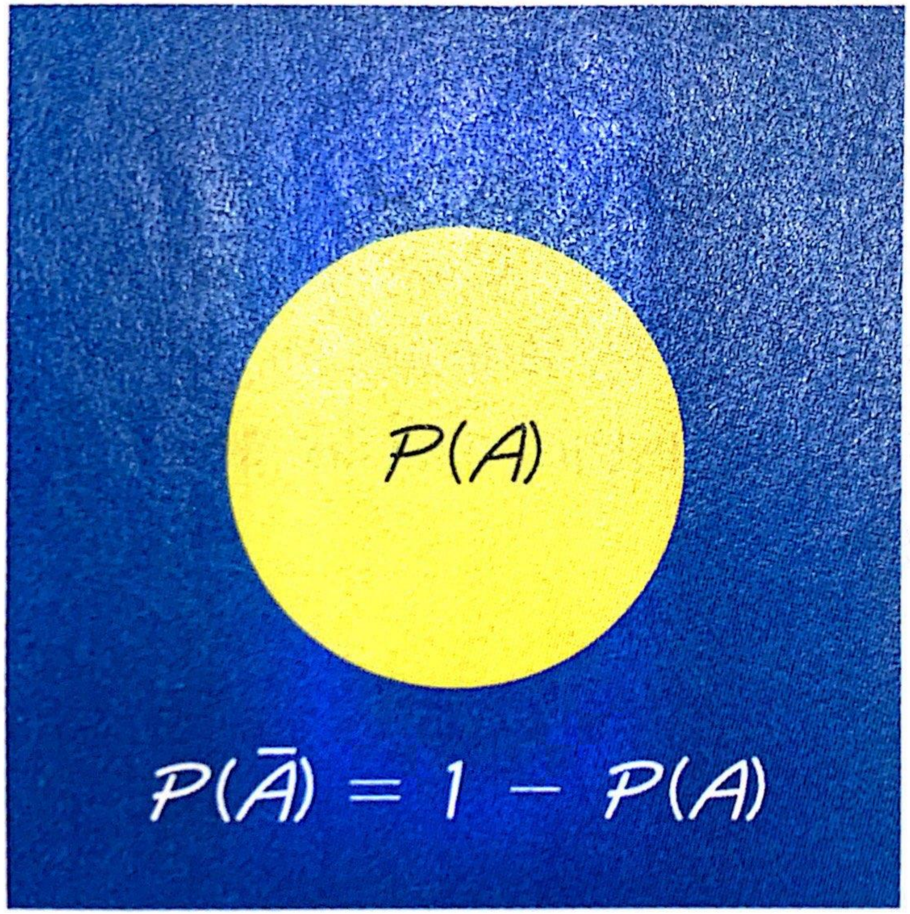


Figure 4-6 **Venn Diagram for the Complement of Event A**

P156 # 40, 42, 43

- If A and B are disjoint and events B and C are disjoint, must events A and C be disjoint?
- Write an expression for $P(A\ or\ B\ or\ C)$. Hint: Draw a Venn Diagram
- Develop a Formula for the probability of not A or B on a single trial $P(\overline{A\ or\ B})$
- Develop a Formula for the probability of not getting A or not getting B on a single Trial $P(\bar{A}\ or\ \bar{B})$

Homework

- P153 #2, 3, 5-12, 27-30

Probability

4-4 The Multiplication Rule

$P(A \text{ and } B)$

- ***OR*** – addition
- ***AND*** – Multiplication
  - The probability that event A ***and*** event B both occur.

- Careful to make sure that the occurrence of Event A does not affect the probability of Event B

- $P(A \text{ and } B) = P(\mathrm{A}) \cdot P(B|A)$
- $P(B|A)$ - the probability of event B occurring after event A has already occurred.

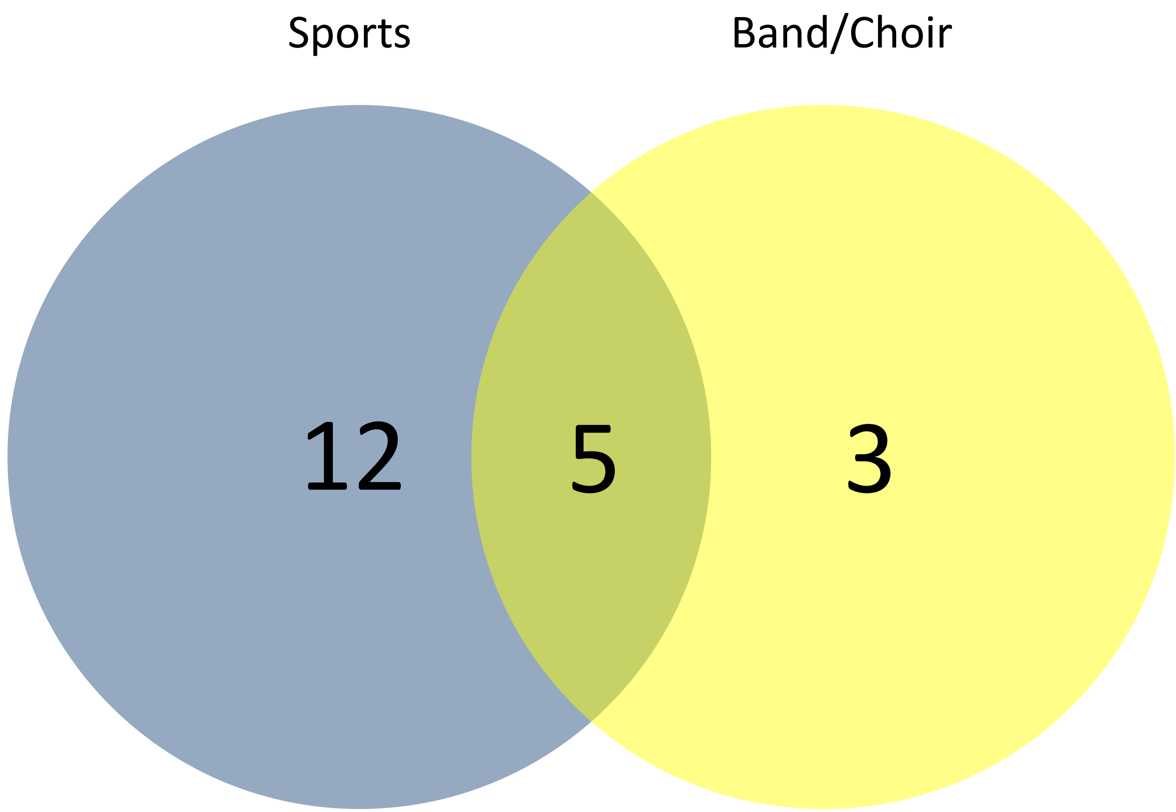
- <u>Independent events</u> – The occurrence of event A does not affect the probability of the occurrence of event B.
- <u>Dependent events</u> – not independent.

- Rolling 2 dice?
- Drawing 2 cards?

Cards

- $P(drawing\ an\ Spade\ and\ an\ Ace)$
- With replacement
- Without replacement

$n = 27$



$$P(Sports) = \frac{17}{27}$$

$$P(Sports\ AND\ Band) = \frac{5}{27} \approx 0.185$$

$$P(Band) = \frac{8}{27}$$

$$\frac{17}{27} \cdot \frac{8}{27} \approx 0.186$$

Rationale for multiplication rule

- POP QUIZ

1. True or False $2 + 2 = 5$
2. Multiple Choice: Mr. Sacco's Coaches:
   a) Football
   b) XC
   c) Soccer
   d) Basketball

- List all possible outcomes in the sample space

Sample Space

- T, a
- T, b
- T, c
- T, d
- F, a
- F, b
- F, c
- F, d

$$P(F\ and\ b) = \frac{1}{8}$$

5% Guideline for Cumbersome Calculations

- When Sample Size is no more than 5% of the size of the population, treat the selections as being independant

Drug Screening

- Use the results from the 50 subjects that used drugs
- Test the probability that we select two drug users with a positive test result
- $P(Positive\ and\ Positive)$

With replacement

$$\frac{44}{50} \cdot \frac{44}{50} \approx 0.7744$$

Without replacement

$$\frac{44}{50} \cdot \frac{43}{49} \approx 0.7722$$

Light Bulbs

- 2400 light bulbs manufactured, 97 are defective
- Test the probability that in a package of 2 that both are defective

With replacement

$$\left(\frac{97}{2400}\right)^2 \approx 0.00163$$

Without

$$\frac{97}{2400} \cdot \frac{96}{2399} \approx 0.00162$$

Light Bulbs

- 2400 light bulbs manufactured, 97 are defective
- Test the probability that in a package of 6 that all 6 are defective

With replacement

$$\left(\frac{97}{2400}\right)^6 \approx 0.00000000435$$

Without

$$\frac{97}{2400} \cdot \frac{96}{2399} \cdot \frac{95}{2398} \cdot \frac{94}{2397} \cdot \frac{93}{2396} \cdot \frac{92}{2395} \approx 0.00000000374$$

Homework

- P164 #5-16, 18, 20, 21

Probability

4-5: Multiplication: Complements and Conditional Probability

The Probability of "At Least One"

- The probability of "at least one" is the same as saying "1 or more"
- At least one LITERALLY means the complement of "none"

At Least One

- Light Bulbs
- 2400 light bulbs manufactured, 97 are defective
- In a 6 pack, what is the probability that At least one is defective
- $1 - P(no\ defective)$

$$\left(\frac{2303}{2400}\right)^6 \approx 0.781$$

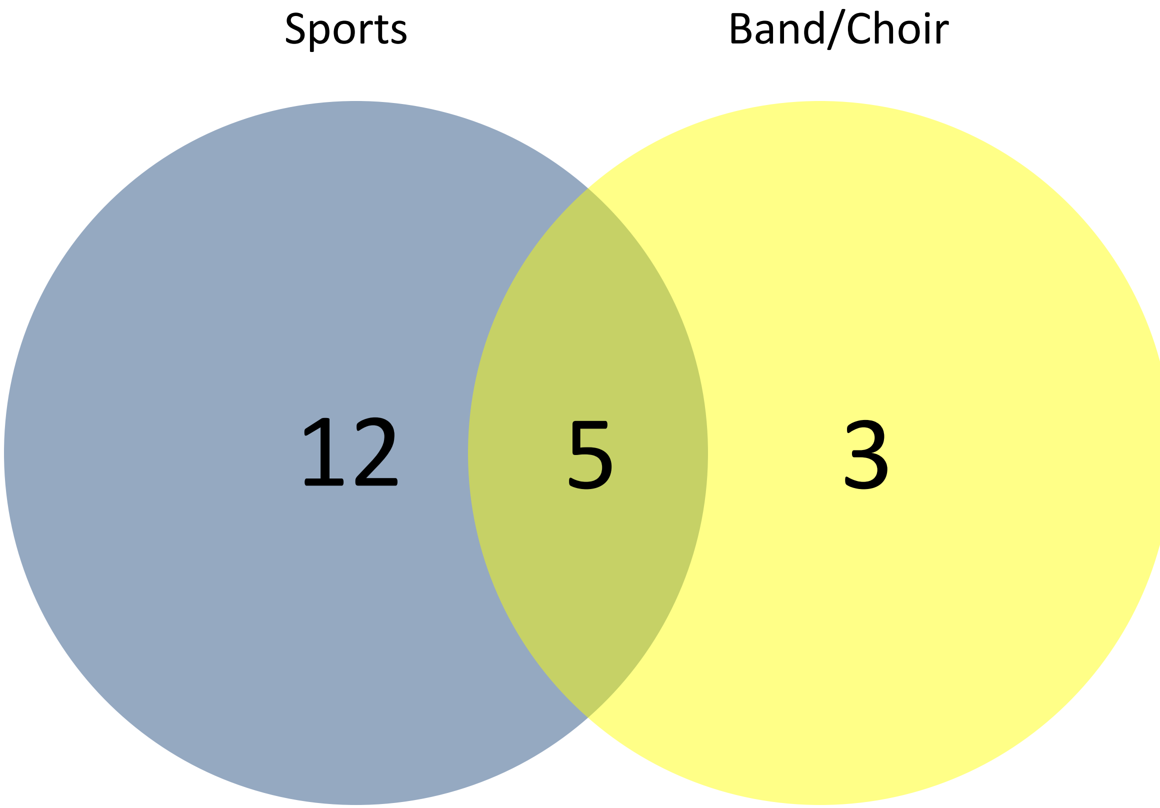
$$1 - 0.781 \approx 0.219$$

Conditional Probability

- $P(B|A) = \frac{P(A\ and\ B)}{P(A)}$

- Examples
  - The probability of selecting a band member given that they are in a sport
    - $P(Band|Sport)$
  - The probability of selecting a sports member given that they are in the band
    - $P(Sports|Band)$

$n = 27$



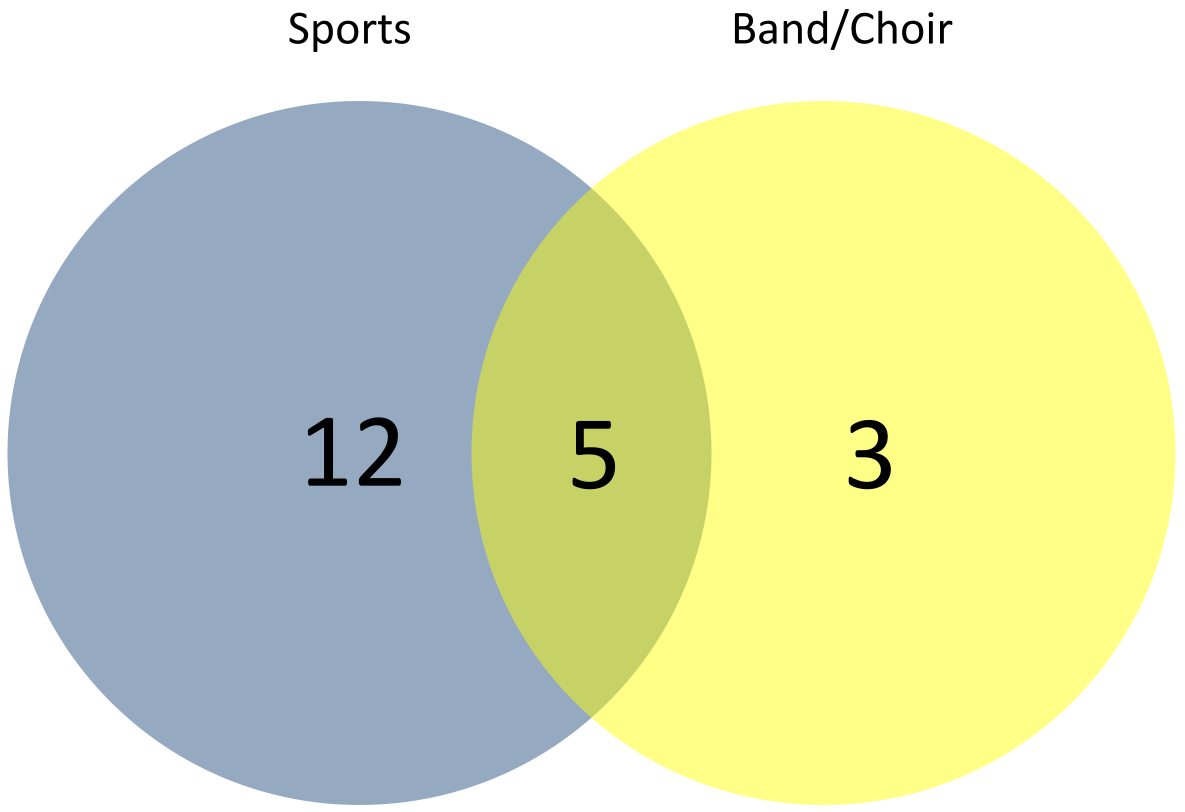
Intuitive Approach

$$P(Sports|Band) = \frac{5}{8}$$

Formula Approach

$$P(Sports|Band) = \frac{5/27}{8/27}$$

$n = 27$



Intuitive Approach

$$P(Band|Sports) = \frac{5}{17}$$

Formula Approach

$$P(Band|Sports) = \frac{5/27}{17/27}$$

$$P(Sport|Band) \neq P(Band|Sport)$$

Homework

- P 172 #5-10, 15, 16, 19, 20, 23-26

Probability

4-6 Counting:

Permutations & Combinations

Counting

- Review: 4-2 – 4-5, avoid formulas
- 4-6: large sums
- Permutations
- Combinations

Permutations v Combinations

- Permutation - arrangements in which different sequences of the same items are counted separately.
- Combinations – arrangements in which different sequences of the same items are ***not*** counted separately.

Permutations v Combinations

- **P**ermutation **P**osition
- **C**ombination **C**ommittee

Fundamental Counting Rule

- List these 6 math teachers in order from youngest to oldest: Belby, Pischke, Spelhaug, Pitcher, Clark, Paustian
- How many arrangements?
- How many ways to get this right?
- Probability of being right?

$$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

Fundamental Counting Rule

- If a license plate follows the rule of 3 letters followed by 4 numbers, how many different license plates can be printed?
- If a license plate can have 7 character (numbers or letters) printed in any order, how many different license plates can be printed?

$$P(A) = \frac{number\ of\ ways\ A\ occurs}{number\ of\ outcomes\ in\ the\ sample\ space}$$

- 11 math teachers sign up for *Run with Carl*. In how many different ways can the math teachers finish 1^{st}, 2^{nd}, & 3^{rd}?

- What is the probability of this finishing order?
  - Belby 1^{st}
  - Pitcher 2^{nd}
  - Spelhaug 3^{rd}

- 3 of the math teachers are selected to be on a special committee. How many different 3 person committees can we make?
- What is the probability of selecting Belby, Pitcher & Spelhaug?

- One hundred people purchase raffle tickets. Three winning tickets will be selected at random. If first prize is \$100, second prize is \$50, and third prize is \$10, in how many different ways can the prizes be awarded?

Combination v. permutation

- Permutations are an *ordered* list
  - Order is important
  - Selected objects should be treated differently
  - Key words – order or arrangement
- Combinations are *unordered* lists
  - The order of selection is irrelevant
  - Selected objects are treated the same
  - Key word – group

$${}_nP_r = \frac{n!}{(n-r)!}$$

Total number of elements in the set

"n Permutate r"

or

"n arrange r"

Number of elements not selected

$$_nC_r = \frac{n!}{r!\,(n-r)!}$$

Total number of elements in the set

"n choose r"

Number of elements *selected*

Number of elements *not* selected

Permutation (identical items)

- How many ways to arrange the letters in the word: SPARTANS

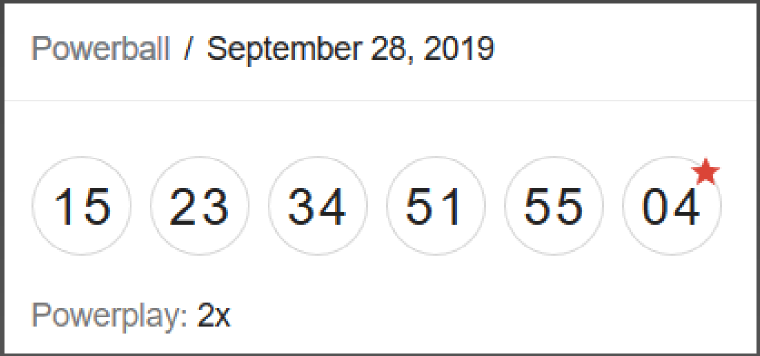
$$\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1) \cdot (2 \cdot 1)}$$

Permutation (identical items)

- How many ways to arrange the letters in the word: MISSISSIPPI

$$\frac{11!}{4!\,4!\,2!}$$

Powerball



- Pick 5 numbers 1 to 69
- Powerball 1 to 26

- Pick 5 numbers 1 to 59
- Powerball 1 to 39

CharityStudio $20 $20

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This detachable stub is your receipt. It contains your unique access code that allows you to download digital music from up-and-coming artists who have teamed up with CharityMania to help non-profit organizations raise money! See the back of this stub to view additional details. Access your music now at the CharityStudio website at www.charitystudio.com.

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308 total prizes totalling more than...

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$600 Highest Valued Card of the Season

$250 Lowest Valued Card of the Season

Top 15 win each week!

$400 Highest Total Points Scored

$100 2nd Highest

$50 3rd Highest

$25 Runner Up Prizes! Next 12 Highest

The bottom 3 also win!

$50 Lowest Total Score

$25 Runner Up Prizes! Next 2 Lowest

FootballMania® is an exciting sweepstakes based on pro football with 18 prizes awarded each week and 2 grand prizes at the end of the season. Each game card has a 1 in 16.6 chance of winning. See the back of this card for sweepstakes rules and the teams you should root for! This sweepstakes is free. No purchase necessary to play.

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Check your game card #
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Your 2019 FootballMania teams by week

Week #1	Week #2	Week #3	Week #4
Dallas NY Jets Washington	Denver NY Giants Tennessee	Detroit Kansas City New Orleans	Green Bay Jacksonville New England
Week #5	**Week #6**	**Week #7**	**Week #8**
Baltimore Carolina Cleveland	Buffalo Cincinnati Miami	Atlanta Chicago LA Rams	Arizona LA Chargers Seattle
Week #9	**Week #10**	**Week #11**	**Week #12**
Chicago Indianapolis Tampa Bay	Houston Oakland San Francisco	Minnesota Philadelphia Pittsburgh	Carolina Denver Pittsburgh
Week #13	**Week #14**	**Week #15**	**Week #16**
Dallas NY Jets San Francisco	Detroit Green Bay LA Chargers	Cincinnati Minnesota NY Giants	Cleveland Houston LA Rams
Week #17	Buffalo, Seattle, Tampa Bay		

FootballMania Sweepstakes Rules

[1] Your game card contains 17 different 3-team combinations, each randomly generated and randomly assigned to the 17 weeks of the 2019 pro football season, scheduled to begin on 9/5/2019. [2] Prizes are awarded each week to the 15 game cards whose teams score the most total combined points relative to all other cards that week. The 3 cards that score the least total points also receive prizes. Grand Prizes are awarded to the one card with the most, and one card with the least, total combined points scored over all 17 weeks. [3] If two or more cards are tied with the same score, 'total net yards' is used as the primary tiebreaker. See website for tiebreaker examples. [4] Football teams that do not play (have a bye) in a given week are assigned that team's score from the previous week. [5] Minimum odds of winning: 1 in 16.6 for the entire season; 1 in 276 in each week; 1 in 2480 for a grand prize. [6] No purchase necessary to play. Void where prohibited. [7] Sweepstakes ends on 12/29/2019. [8] To enter the sweepstakes, the organization on the front of this card must activate the card number. To request a free game card, ask the organization in person (see contact info on front of card) for a "free game card request form", complete the form, and mail to the address shown on the request form along with a self-addressed stamped envelope postmarked by 10/26/2019. A game card will be assigned to you and mailed in the return envelope sent with your form. [9] See website for additional details and to view winning game cards.

Homework

- P180 #5-10, 13-16, 21, 34

Seating chart

- How many ways can I arrange this pod with a class of 28 students?
- How many different 4 person pods can I make from the students that class?

Seating chart

- In a class with 17 girls and 11 boys, what is the probability that this pod will be all girls?
- What is the probability that this pod will have 3 girls and 1 boy?
- What is the probability that this pod will have 2 girls and 2 boys?